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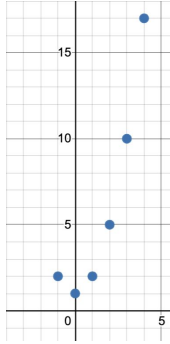



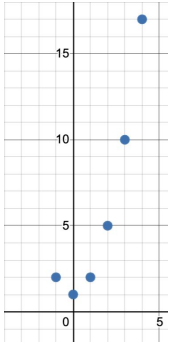



Students who are registered for Grade 12 Advanced Functions (MHF4U) may benefit from a self evaluation and review of the following sample of expectations from Grade 11 Functions (MCR3U) and Grade 12 College Tech Math (MCT4C).










The questions in this self-assessment reflect some of the key ideas learned in prerequisite courses. They do not represent the problem solving approach or the rich experience that students would be exposed to in a classroom. The intention is for students to revisit some key concepts and, if needed, access review materials in an informal environment at a pace that is comfortable for the student.







Concept(s)	Sample Question	How comfortable do you feel with this concept?	Link(s) to explore concept further
<p>I can represent linear and quadratic functions using function notation, given their equations, tables of values, or graphs, and substitute into and evaluate functions</p>	<p>1. Evaluate $f\left(\frac{1}{2}\right)$, given $f(x) = 2x^2 + 3x - 1$.</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Function Notation</p>
<p>I can sketch graphs of $y = af(k(x-d)) + c$ by applying one or more transformations to the graphs of $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \frac{1}{x}$</p>	<p>2. Sketch the graph of $h(x) = 2\sqrt{3(x+1)} - 4$.</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Representing Functions</p>







<p>I can state the domain and range of the transformed function $y = af(k(x - d)) + c$ where $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \frac{1}{x}$</p>	<p>3. State the domain and range of $h(x) = 2\sqrt{3(x+1)} - 4$.</p>	<p> <input type="checkbox"/> Very comfortable</p> <p> <input type="checkbox"/> Somewhat comfortable</p> <p> <input type="checkbox"/> Not at all comfortable</p>	<p>Domain and Range of Two New Functions video</p>
<p>I can determine the algebraic representation of a quadratic function, given the real roots of the corresponding quadratic equation and a point on the function</p>	<p>4. Determine the equation of the quadratic function having zeros at $x=3$ and $x=7$ and passes through the point (2,5).</p>	<p> <input type="checkbox"/> Very comfortable</p> <p> <input type="checkbox"/> Somewhat comfortable</p> <p> <input type="checkbox"/> Not at all comfortable</p>	<p>Families of Parabolas video</p>
<p>I can state the restrictions on the variable values in rational expressions</p>	<p>5. State the restrictions on the following function: $f(x) = \frac{1}{(x+3)(2x-1)}$</p>	<p> <input type="checkbox"/> Very comfortable</p> <p> <input type="checkbox"/> Somewhat comfortable</p> <p> <input type="checkbox"/> Not at all comfortable</p>	<p>Graphical Reciprocal video</p>

<p>I can evaluate numeric expressions containing integer and rational exponents and rational bases</p>	<p>6. Simplify:</p> $\frac{(243x^{-10}y^{15})^{\frac{3}{5}}}{9x^4y^{-5}}$	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Exponents</p>
<p>I can solve exponential equations in one variable by determining a common base</p>	<p>7. Solve the equation $4^{-x} = 8^{x+3}$.</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Comparing Exponential Functions video</p>
<p>I can solve problems using given graphs or equations of exponential functions</p>	<p>8. The number of bacteria in a culture is doubling every 3.75 hours. How long will it take for the number of bacteria to increase from 30 000 to 7 680 000?</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Modelling with Exponential Functions</p>

<p>I can make connections between sequences and discrete functions, represent sequences using function notation</p>	<p>9. Find an equation, in function notation, to represent the following sequences: a) 3, 5, 7, 9, ... b) 4, 20, 100, 500, ... c)</p> 	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Multiple Representations of Sequences</p>
<p>I can distinguish between a discrete function and a continuous function</p>	<p>10. Classify the following as either discrete or continuous. a)</p>  <p>b)</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Discrete versus Continuous Functions video</p>

<p>I can solve problems, using a scientific calculator, that involve the calculation of the amount, A (also referred to as future value, FV), the principal, P (also referred to as present value, PV), or the interest rate per compounding period, i, using the compound interest formula in the form</p> $A = P(1 + i)^n$	<p>11. A student invests \$900 in a term deposit, at per year, compounded monthly, for 5 years. How much interest will the student earn?</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Compound Interest video</p>
<p>I can determine the values of the sine, cosine, and tangent of angles from 0° to 360°</p>	<p>12. Determine the exact values of the following in a manner that demonstrates your understanding:</p> <p>a) sin b) cos c) tan</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Related and Coterminal Angles video</p>
<p>I can prove simple trigonometric identities</p>	<p>13. Prove:</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Trigonometric Identities video</p>

<p>I can describe key properties of periodic functions arising from real-world applications given a graphical representation (i.e., cycle, domain, range, intercepts, amplitude, period, maximum and minimum values, increasing/decreasing intervals)</p>	<p>14. Consider the table and graph showing average monthly temperature (in degrees Celsius) in Alert, Nunavut.</p> <p>Describe the key properties of this data with respect to periodic functions.</p>	<p> <input type="checkbox"/> Very comfortable</p> <p> <input type="checkbox"/> Somewhat comfortable</p> <p> <input type="checkbox"/> Not at all comfortable</p>	<p>Midline, Amplitude, and Period review</p>
<p>I can sketch graphs of $y = af(k(x-d)) + c$ by applying one or more transformations to the graphs of $f(x) = \sin x$ and $f(x) = \cos x$, and state the domain and range of the transformed functions</p>	<p>15. Sketch the graph of</p>	<p> <input type="checkbox"/> Very comfortable</p> <p> <input type="checkbox"/> Somewhat comfortable</p> <p> <input type="checkbox"/> Not at all comfortable</p>	<p>Graphing Sinusoidal Function video</p>

<p>I can represent a sinusoidal function with an equation, given its graph</p>	<p>16. Determine the equation of the function represented by:</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Determining the Equation of a Trig Function video</p>
<p>I can solve problems based on applications involving a sinusoidal function by using a given graph or a graph generated with technology from a table of values or from its equation</p>	<p>17. On a certain day, the depth of water at high tide was 6m above sea level. After 6h, the depth of water was 6m below sea level at a depth of 2m. Assume a 12-h cycle with water at sea level at midnight and the tide is coming in.</p> <p>a) Verify that $h(t)$ models this situation, where</p> $h(t) = 6\sin\frac{\pi}{6}(t) + 8$ <p>b) For how long is the water depth higher than 12m in one day?</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Application of Sinusoidal functions</p>

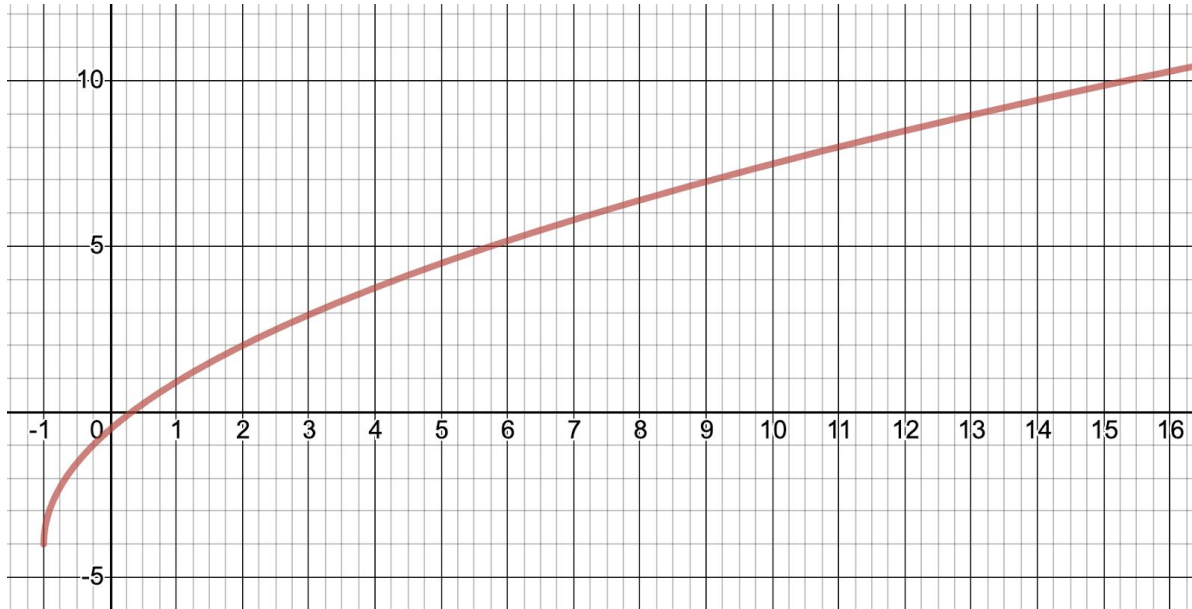
Solutions to Sample Questions

1. Evaluate $f\left(\frac{1}{2}\right)$, given $f(x) = 2x^2 + 3x - 1$.

Solution: $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 1 = 1$

2. Sketch the graph of $h(x) = 2\sqrt{3(x+1)} - 4$

Solution: vertical stretch by factor of 2. Horizontal compression by factor of 3. Slide left 1. Slide down 4.



3. State the domain and range of $h(x) = 2\sqrt{3(x+1)} - 4$.

Solution: $D: \{x \mid x \geq -1, x \in \mathfrak{R}\}$ $R: \{y \mid y \geq -4, y \in \mathfrak{R}\}$

4. Determine the equation of the quadratic function having zeros at $x=3$ and $x=7$ and passes through the point $(2,5)$.

Solution:

$$f(x) = a(x-3)(x-7)$$

$$5 = a(2-3)(2-7)$$

$$a = 1$$

$$\text{Therefore } f(x) = (x-3)(x-7)$$

5. State the restrictions on the following function: $f(x) = \frac{1}{(x+3)(2x-1)}$

Solutions:

$$x+3 \neq 0$$

$$x \neq -3$$

$$2x+1 \neq 0$$

$$2x \neq -1$$

$$x \neq -\frac{1}{2}$$

$$x \neq -3, \frac{1}{2}$$

6. Simplify: $\frac{(243x^{-10}y^{15})^{\frac{3}{5}}}{9x^4y^{-5}}$

Solution:

$$\begin{aligned}
& \frac{(243x^{-10}y^{15})^{\frac{3}{5}}}{9x^4y^{-5}} \\
&= \frac{(\sqrt[5]{243^3x^{-6}y^9})}{9x^4y^{-5}} \\
&= \frac{27x^{-6}y^9}{9x^4y^{-5}} \\
&= 3x^{-10}y^{14} \\
&= \frac{3y^{14}}{x^{10}}
\end{aligned}$$

7. Solve the equation $4^{-x} = 8^{x+3}$.

Solution:

$$4^{-x} = 8^{x+3}$$

$$(2^2)^{-x} = (2^3)^{x+3}$$

$$2^{-2x} = 2^{3x+9}$$

$$-2x = 3x + 9$$

$$-5x = 9$$

$$x = -\frac{9}{5}$$

8. The number of bacteria in a culture is doubling every 3.75 hours. How long will it take for the number of bacteria to increase from 30 000 to 7 680 000?

Solution:

$$7680000 = 30000(2)^{\frac{t}{3.75}}$$

$$25.6 = (2)^{\frac{t}{3.75}}$$

$$25.6^{3.75} = 2^t$$

$$190941 = 2^t$$

$$2^{17.54} = 2^t$$

$$17.54 = t$$

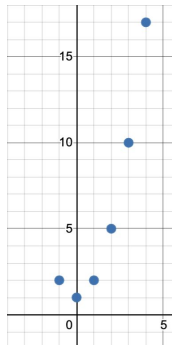
Therefore it will take 17.54 hours for this scenario to take place.

9. Find an equation, in function notation, to represent the following sequences:

a) 3, 5, 7, 9, ...

b) 4, 20, 100, 500, ...

c)



Solutions:

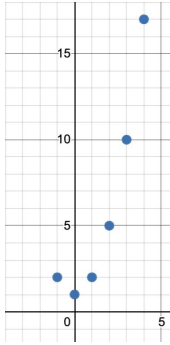
a) $f(x) = 2x + 1$

b) $f(x) = 4(5)^{x-1}$

c) $f(x) = x^2 + 1$

10. Classify the following as either discrete or continuous.

a)



b)

Solutions:

a) Discrete

b) Continuous

11. A student invests \$900 in a term deposit, at _____ per year, compounded monthly, for 5 years. How much interest will the student earn?

Solution:

$$\begin{aligned}
 &= 900 \left(1 + \frac{0.035}{12} \right)^{60} - 900 \\
 &= 1071.85 - 900 \\
 &= 171.85
 \end{aligned}$$

12. Determine the exact values of the following in a manner that demonstrates your understanding:

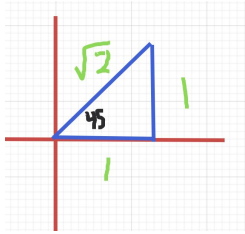
a) sin

b) cos

c) tan

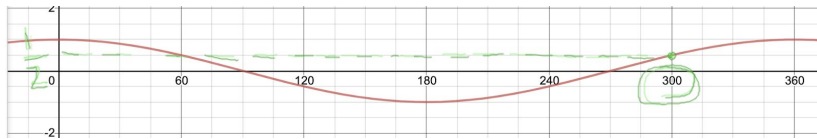
Solution:

a)



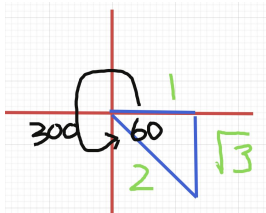
$$\sin 45 = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}}$$

b)



Using interpolation on the graph, it can be seen that $\cos 120 = \frac{1}{2}$.

c)



$$\tan 300 = \frac{\text{opp}}{\text{adj}} = \frac{-2}{\sqrt{3}}$$

13. Prove:

Solution:

L.S.

R.S. = $\cot^2 x$

$$\begin{aligned} &= \frac{\sin^2x + \cos^2x + \cot^2x}{1 + \tan^2x} \\ &= \frac{1 + \cot^2x}{1 + \tan^2x} \\ &= \frac{\csc^2x}{\sec^2x} \\ &= \frac{1}{\frac{\sin^2x}{1}} \\ &= \frac{1}{\cos^2x} \\ &= \frac{\cos^2x}{\sin^2x} \\ &= \cot^2x \\ &= R.S. \end{aligned}$$

Since L.S. = R.S.

Therefore

14. Consider the table and graph showing average monthly temperature (in degrees Celsius) in Alert, Nunavut

Describe the key properties of this data with respect to periodic functions.

Solution:

For Average Max Temperature:

Maximum: 3.3

Minimum: -33.4

Period length: 12 months

Amplitude: 15.05

Phase Shift: for the cosine graph, there is no shift

For the sine graph, the shift is approx 4 months to the right

Cycle: 12 months

Domain: $D: \{m \mid m > 0, m \in \mathbb{Z}\}$

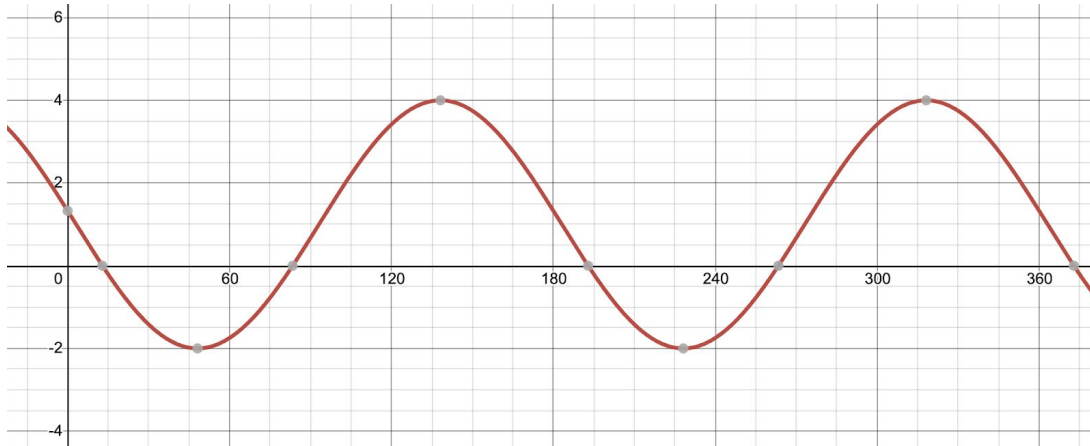
Range: $D: \{t \mid -33.4 \leq t \leq 3.3, t \in \mathfrak{R}\}$

Increasing: $2 < m < 7$

Decreasing: $1 < m < 2, 7 < m < 12$

15. Sketch the graph of .

Solution:



16. Determine the equation of the function represented by:

Solution:

$$f(x) = 2\sin\frac{1}{2}(x + 45) + 1$$

17. On a certain day, the depth of water at high tide was 6m above sea level. After 6h, the depth of water was 6m below sea level at a depth of 2m. Assume a 12-h cycle with water at sea level at midnight and the tide is coming in.

a) Verify that $h(t)$ models this situation, where $h(t) = 6\sin 30(t) + 8$.

b) For how long is the water depth higher than 12m in one day?

Solutions:

a) Some key details from the problem include:

Maximum: 14

Minimum: 2

Period length: 12 hours assists to find the k value.

$$12 = \frac{360}{k}$$

$$k = \frac{360}{12}$$

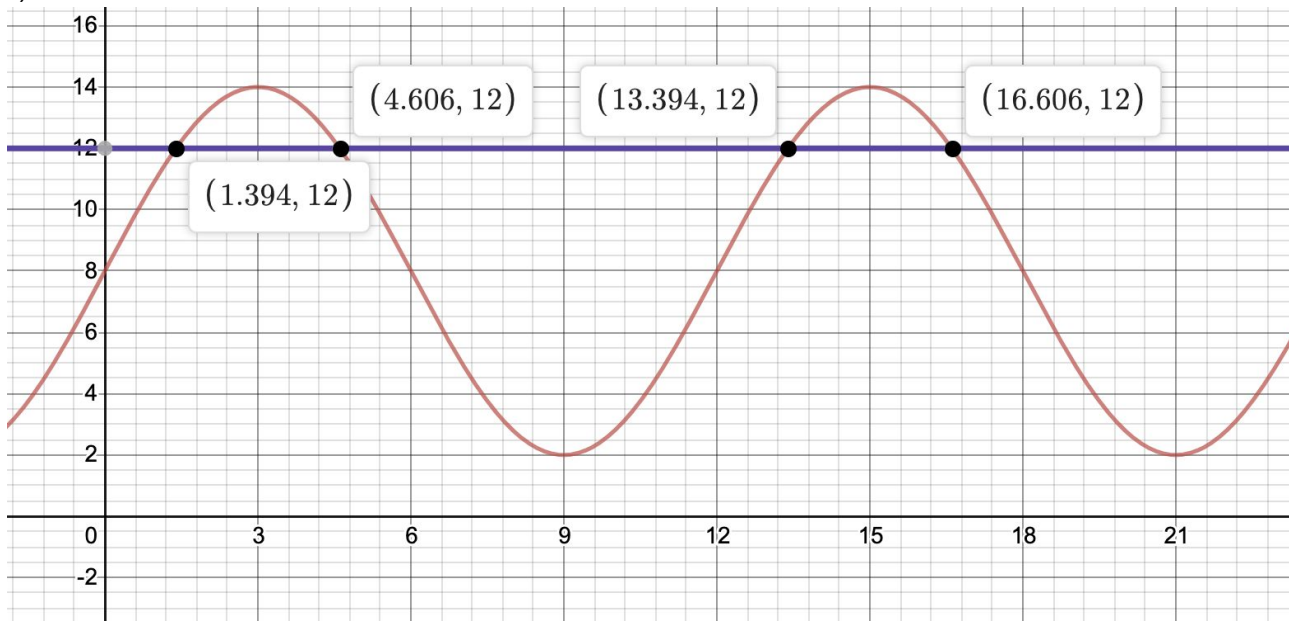
$$k = 30$$

Amplitude: 6

Vertical translation: 8

Phase Shift: Since at midnight $t=0$, and the height is given as at sea level, there is not a phase shift to represent

b)



$$12 = (4.606 - 1.394) + (16.606 - 13.394) = 6.424$$

Therefore the time that the height is greater than 12m is 6.4 hours.