Self-Assessment for Grade 12 Advanced Functions (MHF4U)

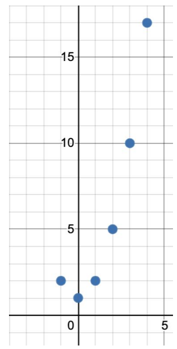
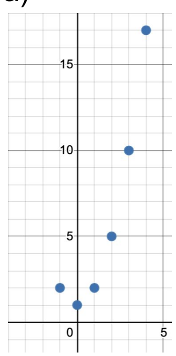
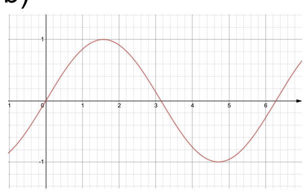
Students who are registered for Grade 12 Advanced Functions (MHF4U) may benefit from a self evaluation and review of the following sample of expectations from Grade 11 Functions (MCR3U) and Grade 12 College Tech Math (MCT4C).

The questions in this self-assessment reflect some of the key ideas learned in prerequisite courses. They do not represent the problem solving approach or the rich experience that students would be exposed to in a classroom. The intention is for students to revisit some key concepts and, if needed, access review materials in an informal environment at a pace that is comfortable for the student.

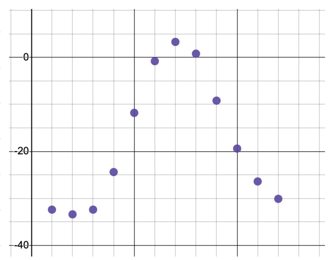
Concept(s)	Sample Question	How comfortable do you feel with this concept?	Link(s) to explore concept further
I can represent linear and quadratic functions using function notation, given their equations, tables of values, or graphs, and substitute into and evaluate functions	1. Evaluate $f\left(\frac{1}{2}\right)$, given $f(x) = 2x^2 + 3x - 1$.	 ☐ **Very comfortable**  ☐ **Somewhat comfortable**  ☐ **Not at all comfortable**	[Function Notation]()
I can sketch graphs of $y = af(k(x-d)) + c$ **by applying one or more transformations to the graphs of** $f(x) = x$**,** $f(x) = x^2$**,** $f(x) = \sqrt{x}$**, and** $f(x) = \frac{1}{x}$	2. Sketch the graph of $h(x) = 2\sqrt{3(x+1)} - 4$.	 ☐ **Very comfortable**  ☐ **Somewhat comfortable**  ☐ **Not at all comfortable**	[Representing Functions]()

I can state the domain and range of the transformed function $y = af(k(x-d)) + c$ **where** $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$ **, and** $f(x) = \frac{1}{x}$	3. State the domain and range of $h(x) = 2\sqrt{3(x+1)} - 4$.	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Domain and Range of Two New Functions video]()
I can determine the algebraic representation of a quadratic function, given the real roots of the corresponding quadratic equation and a point on the function	4. Determine the equation of the quadratic function having zeros at $x=3$ and $x=7$ and passes through the point (2,5).	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Families of Parabolas video]()
I can state the restrictions on the variable values in rational expressions	5. State the restrictions on the following function: $f(x) = \dfrac{1}{(x+3)(2x-1)}$	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Graphical Reciprocal video]()

I can evaluate numeric expressions containing integer and rational exponents and rational bases	6. Simplify: $$\frac{\left(243x^{-10}y^{15}\right)^{\frac{3}{5}}}{9x^{4}y^{-5}}$$	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Exponents]
I can solve exponential equations in one variable by determining a common base	7. Solve the equation $4^{-x}=8^{x+3}$.	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Comparing Exponential Functions video]
I can solve problems using given graphs or equations of exponential functions	8. The number of bacteria in a culture is doubling every 3.75 hours. How long will it take for the number of bacteria to increase from 30 000 to 7 680 000?	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Modelling with Exponential Functions]

I can make connections between sequences and discrete functions, represent sequences using function notation	9. Find an equation, in function notation, to represent the following sequences: a) 3, 5, 7, 9, ... b) 4, 20, 100, 500, … c) 	 ☐ **Very comfortable**  ☐ **Somewhat comfortable**  ☐ **Not at all comfortable**	[Multiple Representations of Sequences]()
I can distinguish between a discrete function and a continuous function	10. Classify the following as either discrete or continuous. a)  b) 	 ☐ **Very comfortable**  ☐ **Somewhat comfortable**  ☐ **Not at all comfortable**	[Discrete versus Continuous Functions video]()

I can solve problems, using a scientific calculator, that involve the calculation of the amount, A (also referred to as future value, FV), the principal, P (also referred to as present value, PV), or the interest rate per compounding period, i, using the compound interest formula in the form $A = P(1 + i)^n$	11. A student invests $900 in a term deposit, at 3.5% per year, compounded monthly, for 5 years. How much interest will the student earn?	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Compound Interest video]
I can determine the values of the sine, cosine, and tangent of angles from $0°$ **to** $360°$	12. Determine the exact values of the following in a manner that demonstrates your understanding: a) sin $45°$ b) cos $120°$ c) tan $300°$	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Related and Coterminal Angles video]
I can prove simple trigonometric identities	13. Prove: $\dfrac{\sin^2 x + \cos^2 x + \cot^2 x}{1 + \tan^2 x} = \cot^2 x$	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Trigonometric Identities video]

<table>
<tr>
<td>I can describe key properties of periodic functions arising from real-world applications given a graphical representation (i.e., cycle, domain, range, intercepts, amplitude, period, maximum and minimum values, increasing/decreasing intervals)</td>
<td>14. Consider the table and graph showing average monthly temperature (in degrees Celsius) in Alert, Nunavut.
<table>
<tr><th>m</th><th>t</th></tr>
<tr><td>1</td><td>−32.4</td></tr>
<tr><td>2</td><td>−33.4</td></tr>
<tr><td>3</td><td>−32.4</td></tr>
<tr><td>4</td><td>−24.4</td></tr>
<tr><td>5</td><td>−11.8</td></tr>
<tr><td>6</td><td>−0.8</td></tr>
<tr><td>7</td><td>3.3</td></tr>
<tr><td>8</td><td>0.8</td></tr>
<tr><td>9</td><td>−9.2</td></tr>
<tr><td>10</td><td>−19.4</td></tr>
<tr><td>11</td><td>−26.4</td></tr>
<tr><td>12</td><td>−30.1</td></tr>
</table>


Describe the key properties of this data with respect to periodic functions.</td>
<td>☐ Very comfortable
☐ Somewhat comfortable
☐ Not at all comfortable</td>
<td>Midline, Amplitude, and Period review</td>
</tr>
<tr>
<td>I can sketch graphs of $y = af(k(x-d)) + c$ by applying one or more transformations to the graphs of $f(x) = \sin x$ and $f(x) = \cos x$, and state the domain and range of the transformed functions</td>
<td>15. Sketch the graph of $f(x) = -3\sin(2(x-180)) + 1$.</td>
<td>☐ Very comfortable
☐ Somewhat comfortable
☐ Not at all comfortable</td>
<td>Graphing Sinusoidal Function video</td>
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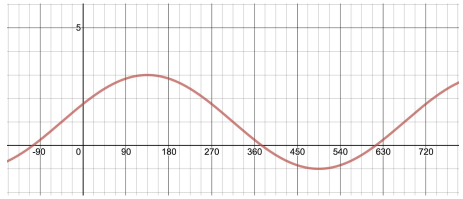
</table>

I can represent a sinusoidal function with an equation, given its graph	16. Determine the equation of the function represented by:  	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	Determining the Equation of a Trig Function video
I can solve problems based on applications involving a sinusoidal function by using a given graph or a graph generated with technology from a table of values or from its equation	17. On a certain day, the depth of water at high tide was 6m above sea level. After 6h, the depth of water was 6m below sea level at a depth of 2m. Assume a 12-h cycle with water at sea level at midnight and the tide is coming in. a) Verify that h(t) models this situation, where $h(t) = 6\sin\frac{\pi}{6}(t) + 8$ b) For how long is the water depth higher than 12m in one day?	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	Application of Sinusoidal functions

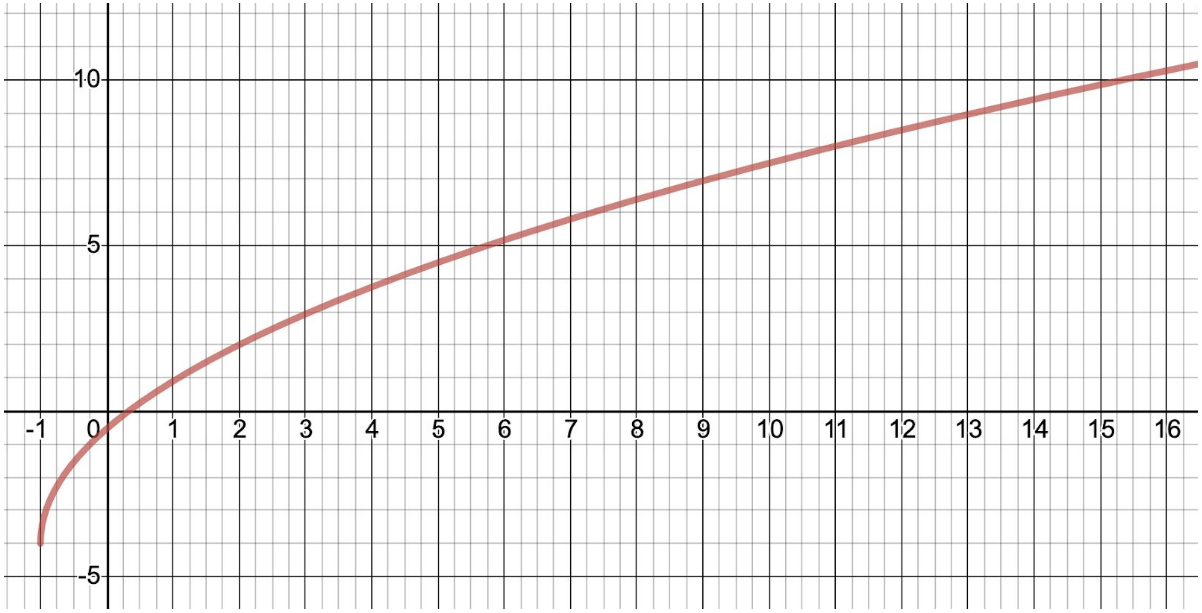
Solutions to Sample Questions

1. Evaluate $f\left(\frac{1}{2}\right)$, given $f(x) = 2x^2 + 3x - 1$.

Solution: $f(\frac{1}{2}) = 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 1 = 1$

2. Sketch the graph of $h(x) = 2\sqrt{3(x+1)} - 4$

Solution: vertical stretch by factor of 2. Horizontal compression by factor of 3. Slide left 1. Slide down 4.



3. State the domain and range of $h(x) = 2\sqrt{3(x+1)} - 4$.

Solution: $D: \{x | x \geq -1, x \in \Re\}$ $R: \{y | y \geq -4, y \in \Re\}$

4. Determine the equation of the quadratic function having zeros at $x = 3$ and $x = 7$ and passes through the point (2,5).

Solution:

$f(x) = a(x-3)(x-7)$

$5 = a(2-3)(2-7)$

$a = 1$

Therefore $f(x) = (x-3)(x-7)$

5. State the restrictions on the following function: $f(x) = \dfrac{1}{(x+3)(2x-1)}$

Solutions:

$x+3 \neq 0$

$x \neq -3$

$2x+1 \neq 0$

$2x \neq -1$

$x \neq -\dfrac{1}{2}$

$x \neq -3, \dfrac{1}{2}$

6. Simplify: $\dfrac{\left(243x^{-10}y^{15}\right)^{\frac{3}{5}}}{9x^{4}y^{-5}}$

Solution:

$$\frac{\left(243x^{-10}y^{15}\right)^{\frac{3}{5}}}{9x^{4}y^{-5}}$$

$$=\frac{\left(\sqrt[5]{243^{3}}x^{-6}y^{9}\right)}{9x^{4}y^{-5}}$$

$$=\frac{27x^{-6}y^{9}}{9x^{4}y^{-5}}$$

$$=3x^{-10}y^{14}$$

$$=\frac{3y^{14}}{x^{10}}$$

7. Solve the equation $4^{-x}=8^{x+3}$.

Solution:

$$4^{-x}=8^{x+3}$$

$$\left(2^{2}\right)^{-x}=\left(2^{3}\right)^{x+3}$$

$$2^{-2x}=2^{3x+9}$$

$$-2x=3x+9$$

$$-5x=9$$

$$x=-\frac{9}{5}$$

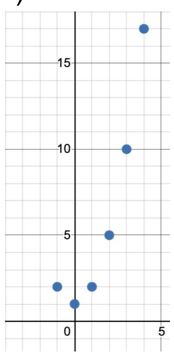
8. The number of bacteria in a culture is doubling every 3.75 hours. How long will it take for the number of bacteria to increase from 30 000 to 7 680 000?

Solution:

$$
\begin{aligned}
7680000 &= 30000(2)^{\frac{t}{3.75}} \\
25.6 &= (2)^{\frac{t}{3.75}} \\
25.6^{3.75} &= 2^{t} \\
190941 &= 2^{t} \\
2^{17.54} &= 2^{t} \\
17.54 &= t
\end{aligned}
$$

Therefore it will take 17.54 hours for this scenario to take place.

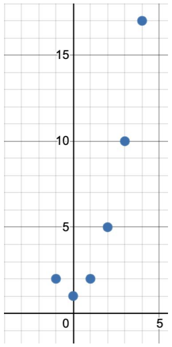
9. Find an equation, in function notation, to represent the following sequences:
a) 3, 5, 7, 9, ...
b) 4, 20, 100, 500, …
c)



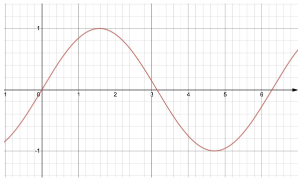
Solutions:

a) $f(x) = 2x + 1$
b) $f(x) = 4(5)^{x-1}$
c) $f(x) = x^2 + 1$

10. Classify the following as either discrete or continuous.
a)



b)



Solutions:
a) Discrete
b) Continuous

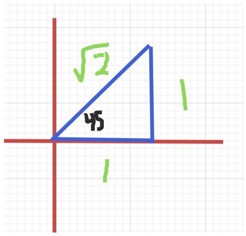
11. A student invests $900 in a term deposit, at 3.5% per year, compounded monthly, for 5 years. How much interest will the student earn?

Solution:

$$
\begin{aligned}
&= 900\left(1+\tfrac{0.035}{12}\right)^{60} - 900 \\
&= 1071.85 - 900 \\
&= 171.85
\end{aligned}
$$

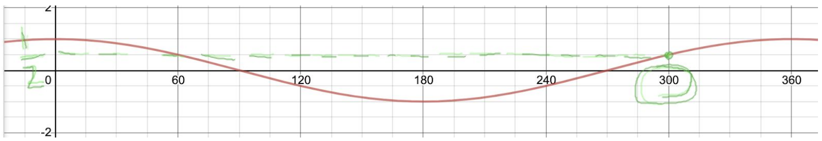
12. Determine the exact values of the following in a manner that demonstrates your understanding:
a) sin $45°$
b) cos $120°$
c) tan $300°$

Solution:
a)



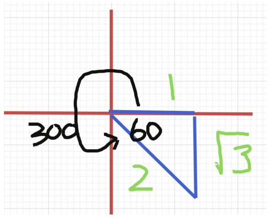
$$\sin 45 = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$$

b)



Using interpolation on the graph, it can be seen that $\cos 120 = \frac{1}{2}$.

c)



$$\tan\ 300 = \frac{opp}{adj} = \frac{-\sqrt{3}}{2}$$

13. Prove: $\dfrac{\sin^2 x + \cos^2 x + \cot^2 x}{1 + \tan^2 x} = \cot^2 x$.

Solution:

L.S.

R.S. = $cot^2 x$

$$=\frac{\sin^2 x+\cos^2 x+\cot^2 x}{1+\tan^2 x}$$

$$=\frac{1+\cot^2 x}{1+\tan^2 x}$$

$$=\frac{csc^2 x}{sec^2 x}$$

$$=\frac{\frac{1}{\sin^2 x}}{\frac{1}{\cos^2 x}}$$

$$=\frac{\cos^2 x}{\sin^2 x}$$

$$=cot^2 x$$

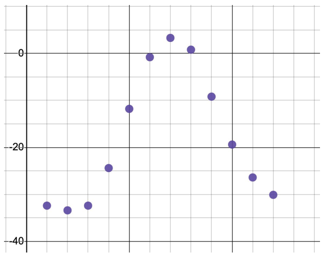
$$=R.S.$$

Since L.S. = R.S.

Therefore $\frac{\sin^2 x+\cos^2 x+\cot^2 x}{1+\tan^2 x}=\cot^2 x$

14. Consider the table and graph showing average monthly temperature (in degrees Celsius) in Alert, Nunavut

m	t
1	−32.4
2	−33.4
3	−32.4
4	−24.4
5	−11.8
6	−0.8
7	3.3
8	0.8
9	−9.2
10	−19.4
11	−26.4
12	−30.1



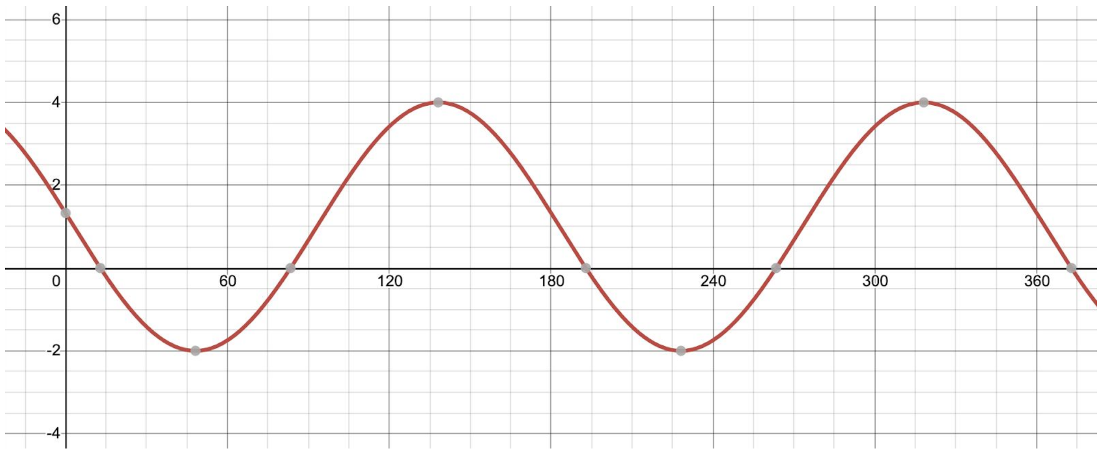
Describe the key properties of this data with respect to periodic functions.

Solution:

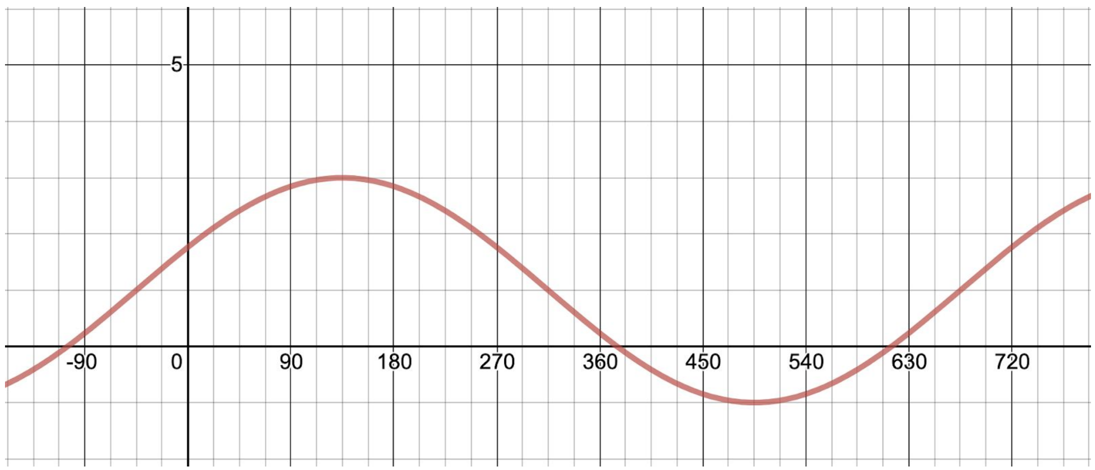
For Average Max Temperature:
Maximum: 3.3
Minimum: -33.4
Period length:12 months
Amplitude: 15.05
Phase Shift: for the cosine graph, there is no shift
For the sine graph, the shift is approx 4 months to the right
Cycle: 12 months
Domain: $D:\{m|m>0, m\in \mathbb{Z}\}$
Range: $D:\{t|-33.4\leq t\leq 3.3, t\in \mathfrak{R}\}$
Increasing: $2 < m < 7$
Decreasing: $1 < m < 2,\ 7 < m < 12$

15. Sketch the graph of $f(x) = -3\sin(2(x-180)) + 1$.

Solution:



16. Determine the equation of the function represented by:



Solution:

$$f(x) = 2\sin\frac{1}{2}(x + 45) + 1$$

17. On a certain day, the depth of water at high tide was 6m above sea level. After 6h, the depth of water was 6m below sea level at a depth of 2m. Assume a 12-h cycle with water at sea level at midnight and the tide is coming in.

a) Verify that h(t) models this situation, where $h(t) = 6\sin 30(t) + 8$.

b) For how long is the water depth higher than 12m in one day?

Solutions:

a) Some key details from the problem include:
Maximum: 14
Minimum: 2
Period length:12 hours assists to find the k value.

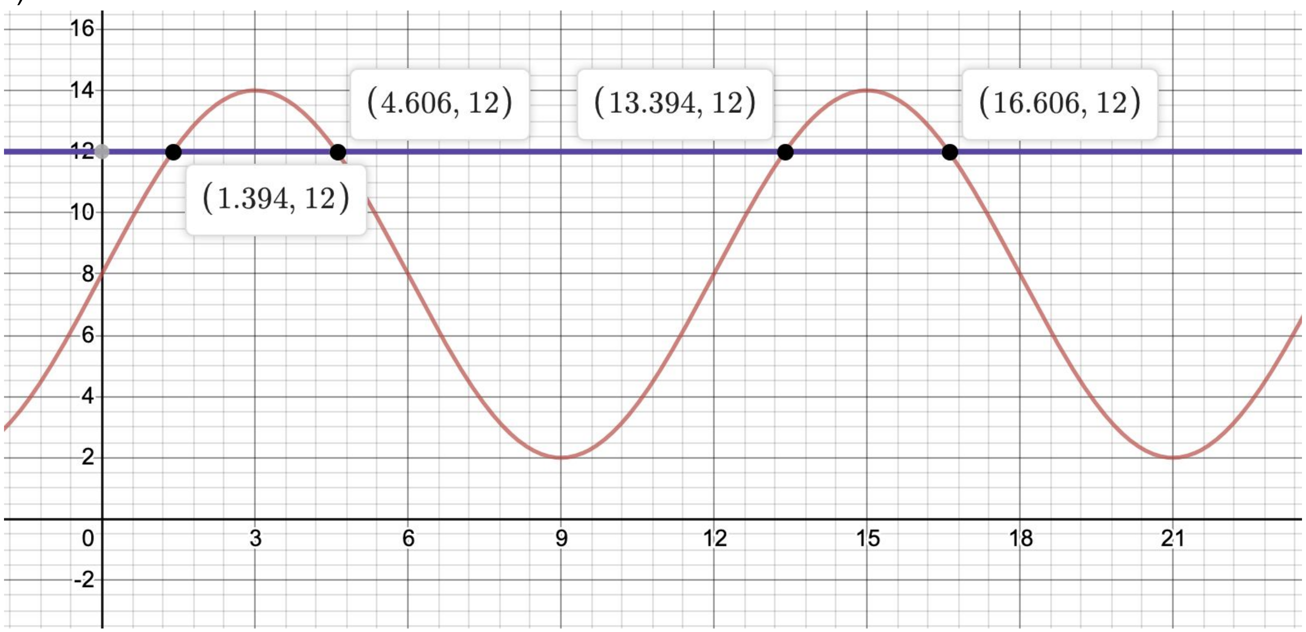
$$12=\frac{360}{k}$$

$$k=\frac{360}{12}$$

$$k=30$$

Amplitude: 6
Vertical translation: 8
Phase Shift: Since at midnight t =0, and the height is given as at sea level, there is not a phase shift to represent

b)



$$12 = (4.606 - 1.394) + (16.606 - 13.394) = 6.424$$

Therefore the time that the height is greater than 12m is 6.4 hours.